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14. ABSTRACT The theme of this research has been to develop effective strategies for the design and analysis of flight control systems for tailless fighter aircraft. In our quest to understand the fundamental issues in the aggressive maneuvering of high performance fighter aircraft, we have developed tools and techniques, and the appropriate supporting theoretical results, for the study and analysis of high performance maneuvering systems.

We have developed a simplified flight dynamics model, the coordinated flight vehicle (CFV). Exploiting the geometric structure of the CFV, one may easily explore the space of aggressive flight maneuvers. We have developed an optimization based, trajectory morphing technique by which CFV maneuvers may be used to parametrize the achieveable flight maneuvers (i.e., maneuvers that satisfy given high fidelity aircraft model dynamics). We have also developed theory and algorithms to allow these model based optimization techniques to used online, in a receding horizon fashion. As a theoretical bonus, we have obtained new results on the structure of the trajectory manifold of a nonlinear system.

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Robust Nonlinear Control of Piloted Tailless Fighters

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The theme of this research has been to develop effective strategies for the design and analysis of flight control systems for tailless fighter aircraft. In our quest to understand the fundamental issues in the aggressive maneuvering of high performance fighter aircraft, we have developed tools and techniques, and the appropriate supporting theoretical results, for the study and analysis of high performance maneuvering systems.

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Aggressive Flight Maneuvers

It is important to evaluate the performance of the overall flight control system of tailless fighters over a class of aggressive maneuvers representative of fighter combat [8]. Although a suite of maneuvers could be supplied by recording the actions of a pilot in simulated flight, the level of effort required to equip a pilot-in-the-loop simulator with appropriate dynamics is difficult to justify in the early stages of flight control systems development and evaluation. Moreover, it is desirable to have methods for exploring the space of maneuvers to identify potential shortcomings in the aircraft or flight control system design.

To this end, we have developed a strategy for the exploration of aggressive flight maneuvers. Our approach makes use of the fact that most aircraft (and pilots) are designed to fly in a coordinated manner (i.e., without sideslip). Explicitly enforcing this as a constraint, we have obtained a simplified flight dynamics model, the coordinated flight vehicle [1], possessing many interesting and useful features. First, the coordinated flight vehicle highlights the rich geometry of flight (e.g., the Hopf fibration of $SO(3)$ pops up). Second, the CFV is a nice control system in that it is differentially flat and path invertible. Finally, the coordinated flight vehicle captures the global nonlinear dynamics of maneuvering flight. It is precisely these dynamics that a pilot strives to manage.

Using the coordinated flight vehicle, we have developed some prototype tools for the exploration and visualization of flight maneuvers. These tools utilize the OpenGL graphics library (as a C/C++ library under Windows 95/NT) to allow 3D dynamic visualization (via animation) of an aircraft in maneuvering flight. The possibilities are quite vast and we are working to develop an intuitive interface for control engineers. One of the advantages of using the coordinated flight vehicle derives from its simplicity—interactive tools can be run on rather modest computer systems (e.g., PCs without graphics accelerator hardware).

One of the tools we have developed allows one to fly the aircraft by merely pointing in the direction of desired motion. The aircraft rolls and climbs as needed to execute the desired maneuver. This is accomplished by exploiting the path following properties of the coordinated flight vehicle. With an appropriate interface, tools such as this can be used to specify (or, more accurately, develop) an aggressive maneuver for evaluation on a tailless fighter.

It is interesting to speculate on the many other potential uses of the coordinated flight vehicle. Indeed, one can imagine using the special geometric structure to guide the development of autonomous and semi-

autonomous flight management systems for managing the global nonlinear dynamics of flight vehicles. In that case, one might truly develop a *nonlinear pilot*.

Now, the maneuvers (or trajectories) that are generated using the coordinated flight vehicle are necessarily somewhat idealized. The side slip angle β is kept identically zero over the entire trajectory. Moreover, other important effects have been intentionally removed from consideration. For instance, the coordinated flight vehicle does not account for the small normal force that accompanies the generation of a pitching moment, eliminating the nonminimum phase component of the normal acceleration response. Most of the low level detail of actuation will also be suppressed. On the other hand, for proper control system analysis of maneuvering aircraft, one should not shrink away from the real world aspects of a complicated flight dynamics model, regardless of how ugly the details may be.

Trajectory Morphing for Nonlinear Systems

As noted above, we are interested in finding trajectories of the complicated *truth* model even though our intuition about the nature of maneuvering may come from a considerably simplified system possessing nice properties. To this end, we have developed a homotopy approach [3] for *morphing* a trajectory of a one system (a simple system) into a trajectory of another system (the *real* system). These ideas will be particularly useful in finding detailed trajectories (including actuator dynamics) for complicated systems such as tailless fighter aircraft.

A simple (academic) example will indicate the key ideas in morphing. The dynamics of an inverted pendulum on a cart are given by

$$\dot{x}^T = f(x, u)^T := (x_2, \sin x_1 + \cos x_2 \, u, x_4, u)^T .$$

where the state vector is $x^T = (\theta, \dot{\theta}, y, \dot{y})^T$ and the control input u is the cart acceleration $\ddot{y}$. Although f looks quite simple, it is not easy to specify the trajectories of this *nonlinear pendulum system*.

In contrast, it is *very* easy to specify, in closed form, *all* of the trajectories of the *linear pendulum system*,

$$\dot{x}^T = f_0(x, u)^T := (x_2, x_1 + u, x_4, u)^T ,$$

given by (Jacobian) linearization about the inverted equilibrium position. Indeed, since time invariant linear systems are *differentially flat*, each trajectory of f_0 can be specified by choosing a C^4 (or higher) function. The flat output for the linear pendulum system is $y = h(z) = z_3 - z_1$. Note that the nonlinear pendulum system is not differentially flat.

Now, the general character of trajectories of these two systems is quite similar provided that the pendulum does not depart too far from the inverted position. Indeed, we know from linearizing considerations that for very small pendulum angles, the nonlinear pendulum system will be able to track the trajectories of the linear pendulum system very well. Approximate tracking only works when the desired trajectory is not too aggressive. There is not much a regulator can do when it is fed the contradictory trajectory information derived from an (intentionally) incorrect model.

The important point here is that, even when approximate tracking is no longer viable, one may be able to find trajectories of the more complicated system that are much like those of the simple system—provided that essential couplings are similar. We exploit this idea and use trajectories of a simple system to parametrize the trajectories of a more complicated system.

We have taken the following approach. Given a trajectory $(x_0(\cdot), u_0(\cdot))$ of the simple system f_0, we seek the trajectory $(x_1(\cdot), u_1(\cdot))$ of $f_1 = f$ that is nearest $(x_0(\cdot), u_0(\cdot))$ in a least squares sense. We solve this problem by tracing out a homotopy path in the space of curves. In particular, $(x_\lambda(\cdot), u_\lambda(\cdot))$, $0 \leq \lambda \leq 1$, is the solution of

$$\text{minimize} \quad \frac{1}{2} \int_0^T \|(x(\tau) - x_0(\tau), u(\tau) - u_0(\tau))\|^2 d\tau$$
$$\text{subject to} \quad \dot{x}(t) = f_\lambda(x(t), u(t)), \qquad t \in [0, T]$$

2

where $f_\lambda = (1 - \lambda)f_0 + \lambda f$ defines a homotopy connecting the systems. The solution for $\lambda = 0$ is trivial. Under reasonable conditions (involving differentiation wrt λ), the homotopy path in trajectory space will be nice guaranteeing a solution for $\lambda = 1$. Moreover, since $(x_\lambda(\cdot), u_\lambda(\cdot))$ depends on λ in a smooth manner, the problem of a good initial condition for the optimization problem all but vanishes. Figure 1 shows the
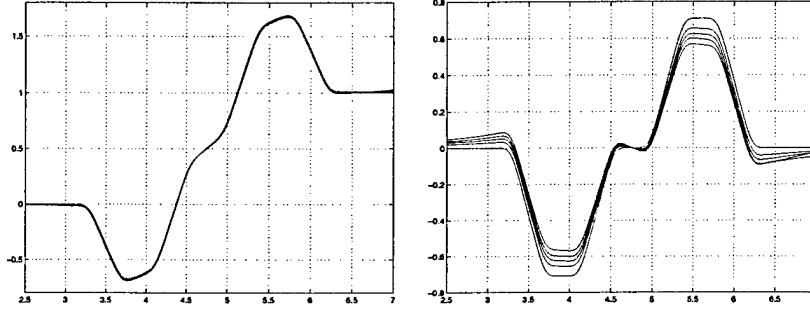

Figure 1: Morphed position and angle trajectories ($\lambda = 0, 0.3, 0.5, 0.7, 1.0$).

morphed position and angle trajectories for the cart and pendulum system(s) executing a sidestep maneuver. Note the angle trajectories for the linear and nonlinear pendulum systems differ significantly. This difference is large enough to make approximate tracking unsuccessful.

The Trajectory Manifold of a Nonlinear Control System

While developing trajectory morphing techiques, it became clear that we really didn't know enough about what the set of trajectories looked like. After all, if we are going to use smooth optimization techniques, wouldn't it be nice if the domain of interest and the functions involved possess some desirable smoothness properties? Searching the literature, we were unable to find results that set our minds to rest.

The object of interest is the set of bounded, exponentially stabilizable trajectories of $\dot{x} = f(x, u)$ which we shall denote by $\mathcal{T}$. Suppose that $\xi_0 = (\alpha_0, \mu_0) \in \mathcal{T}$ (so that $\dot{\alpha}_0(t) = f(\alpha_0(t), \mu_0(t))$) and that $f \in C^r$, $r \geq 1$. Then, there is a time-varying linear feedback K that stabilizes ξ_0. Moreover, the *projection operator* $\mathcal{P} : (\alpha, \mu) \mapsto (x, u)$ defined by

$$
\begin{aligned}
\dot{x}(t) &= f(x(t), u(t)), \qquad x(0) = \alpha(0) \\
u(t) &= \mu(t) + K(t)[\alpha(t) - x(t)]
\end{aligned}
$$

for $t \geq 0$, maps an L_∞ neighborhood of ξ_0 onto the trajectory manifold $\mathcal{T}$. Researchers have used approximate trajectory tracking controllers of this sort for years. It is a very practical tool.

The projection operator $\mathcal{P}$ also turn out to be a very useful theoretical tool [2]. We have shown that

- $\mathcal{P}$ is C^r (wrt to L_∞ norm). Explicit formulas for the derivatives are provided.

- $\mathcal{T}$ is a C^r Banach Manifold.

- $T_\xi \mathcal{T}$ is the set of bounded trajectories of the linearization of f around ξ.

While results of this sort are expected, there are some surprises. For example, $\mathcal{P}$ is **not** differentiable in L_2! With a clearer understanding in hand, we have been able to develop improved algorithms for local optimization on the trajectory manifold, including morphing techniques.

Receding Horizon Control of Nonlinear Systems

The techniques developed above indicate that optimization can play an important role in the planning and control of high performance maneuvering. For instance, once the broad strokes of a maneuver have been outlined using a coordinated flight vehicle model, important details of the low level trajectory for tracking can be obtained using trajectory morphing. To be responsive to environmental factors such as newly discovered threats, it is critical that the optimizations be conducted online, in a receding horizon fashion. Although online optimization has been used extensively in the model predictive control of chemical processes (relatively slow linear dynamics with important constraints), its applicability to highly unstable systems with fast nonlinear dynamics remained a question.

It is well known that unconstrained infinite horizon optimal control may be used to construct a stabilizing controller for a nonlinear system. We have shown [6, 5, 7, 4] that similar stabilization results may be achieved using *unconstrained* finite horizon optimal control. The key idea is to approximate the tail of the infinite horizon cost-to-go using, as terminal cost, an appropriate control Lyapunov function (CLF). Roughly speaking the terminal CLF should provide an (incremental) upper bound on the cost. In this fashion, important stability characteristics may be retained *without* the use of terminal constraints such as those employed by a number of other researchers. The absence of constraints allows a significant speedup in computation. Furthermore, it is shown that in order to guarantee stability, it suffices to satisfy an improvement property, thereby relaxing the requirement that truly *optimal* trajectories be found. We provide a complete analysis of the stability and region of attraction/operation properties of receding horizon control strategies that utilize finite horizon approximations in the proposed class. The guaranteed region of operation contains that of the CLF controller and may be made as large as desired by increasing the optimization horizon (restricted, of course, to the infinite horizon domain). Moreover, it is easily seen that both CLF and infinite horizon optimal control approaches are limiting cases of our receding horizon strategy.
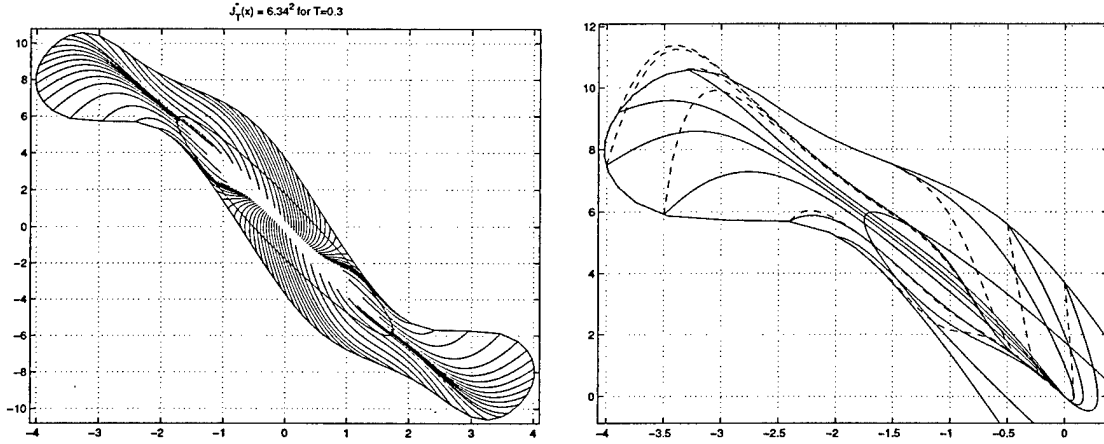

Figure 2: (a) The receding horizon operating region Γ_r^T for horizon length $T = 0.3$ and radius $r = r_v = 6.34$ together with CLF operating region Ω_{r_v}. Also depicted are the trajectories $x_T^*(\cdot; x)$ for x on the boundary of Γ_r^T. (b) Receding horizon $\mathcal{R}(0.3, 0.05)$ and CLF controller (dashed) trajectories.

The key results of this work have been illustrated on a nontrivial nonlinear control system, the inverted pendulum on a cart, where significant improvements in guaranteed region of operation and cost are noted. Figure 2 depicts the regions of operation for the receding horizon strategy and a control Lyapunov function derived control law. By comparing the closed loop trajectories, we can see that the effect of the online optimization is to make better use of system nonlinearities providing significant performance improvements.

4

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